Transpiration by Root Water Uptake

GEO-SLOPE International Ltd. | www.geo-slope.com

1200, 700 - 6th Ave SW, Calgary, AB, Canada T2P 0T8
Main: +1 403 269 2002 | Fax: +1 888 463 2239

Introduction

Transpiration results in water being extracted from the ground by root mass, which in-turn alters the pore-water pressures and therefore the groundwater flow regime. Root water uptake involves a complex interaction between atmospheric climate conditions, vegetation, and the ground surface. The root water uptake boundary condition in SEEP/W calculates the water extraction rates in accordance with potential evapotranspiration, which is climate dependent, the condition of the vegetation, and the availability. This example explores the implementation and definition of the root water uptake boundary condition. Calculations are done to verify the simulated results and a commentary is provided on the role of mesh density.

Background

The land-climate interaction (LCI) boundary condition comprises two components: one for calculating the net infiltration at the ground surface and another for calculating the root water uptake (RWU) within the soil profile. The first component is ultimately concerned with ensuring a water mass balance at the ground surface:

$$(q_P + q_M)cos\alpha + q_E + q_R = q_I$$ **Equation 1**

where superscripts on the water fluxes (q) indicate rainfall (P), snow melt (M), infiltration (I), evaporation (E) and runoff (R) and α is the slope angle. Transpiration, the second component of the LCI boundary condition, does not appear in Equation 4 because root water uptake occurs below the ground surface. The potential transpiration flux (PT) is calculated as:

$$q_{PT} = q_{PET}(SCF)$$ **Equation 2**

where SCF is the Soil Cover Fraction that varies from zero to one for bare ground to full coverage conditions, respectively. Various expression exist in the literature for calculating the SCF as a function of leaf area index (LAI). The LAI is a dimensionless quantity that characterizes plant canopies as the one-sided green leaf area per unit ground surface area. The LAI can be defined as a function of time.

The maximum possible root water extraction rate per volume of soil q_{root}^{max} ($L^3/t/L^3$) at any particular depth can be calculated from the potential transpiration flux q_{PT} as (Feddes et al., 2001):

$$q_{root}^{max} = \pi_{root}^{'} q_{PT}$$ **Equation 3**

where $\pi_{root}^{'}$ is the normalized water uptake distribution (L^{-1}). The actual root water uptake is less than the maximum due to stresses such as dry or wet conditions and high salinity concentrations (i.e. osmotic suctions). Wet conditions produce oxygen deficiency and dry conditions limit the availability of water. The water availability and salinity stresses can be assumed to be multiplicative, in which case the actual root water extraction rate is given by (Feddes et al., 2001):

$$q_{root} = \alpha_{rw} \alpha_{rs} q_{root}^{max}$$ **Equation 4**

where α_{rw} and α_{rs} are the reduction factors due to water and salinity stresses, respectively. The term α_{rw} is defined by a plant limiting function. The LCI boundary condition in SEEP/W does not accommodate salinity stresses. Figure 1 illustrates a simple linear variation in α_{rw} in which the root water extraction rate is limited between matric suctions S1 and S2 due to anaerobic conditions and between S3 and S4 by a reduction in water availability. Matric suction S4 defines the wilting point of the vegetation.

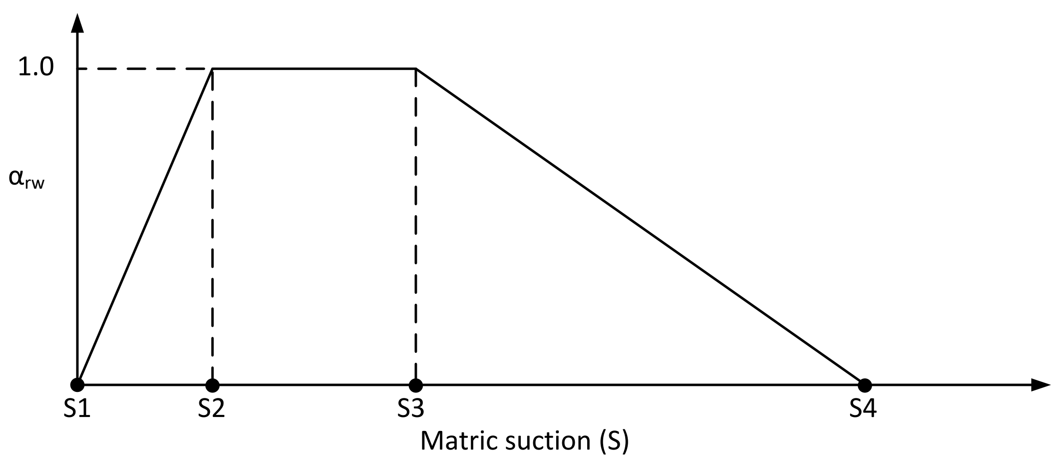

Figure 1. Linear variation of the plant limiting factor α_{rw} with matric suction.

The normalized water uptake distribution $\pi_{root}^{'}$ (L^{-1}) is given as:

$$\pi_{root}' = \frac{\pi_{root}}{\int_0^{r_{max}} \pi_{root} dr}$$ Equation 5

where π_{root} is the root length density (L/L^3); that is, the length of root per volume of soil. Integration of the root density function π_{root} over the maximum root depth r_{max} gives the total root length beneath a unit area (L/L^2). Normalizing the uptake distribution ensures that π_{root}' integrates to unity over the maximum root depth; that is,

$$\int_0^{r_{max}} \pi_{root}' dr = 1.0$$ Equation 6

Integration of Equation 4 over the rooting depth recovers the actual transpiration flux:

$$q_{AT} = \int_0^{r_{max}} q_{root} dr$$ Equation 7

There are a multitude of expressions in the literature to describe the normalized water uptake distribution if the root length density is not measured. Hoffman and van Genuchten (1983) assumed:

$$\pi_{root}' = \begin{cases} \frac{1.667}{r_{max}} & r < 0.2r_{max} \\ \frac{2.0833}{r_{max}}\left(1 - \frac{r}{r_{max}}\right) & 0.2r_{max} \leq r \leq r_{max} \\ 0 & r > r_{max} \end{cases}$$ Equation 8

Prasad (1988) adopted in a one dimensional linear expression:

$$\pi_{root}' = \frac{2}{r_{max}}\left(1 - \frac{r}{r_{max}}\right) \quad r \leq r_{max}$$ Equation 9

Figure 2 compares Equation 8 and Equation 9 and demonstrates an arbitrarily defined normalized water uptake function that would be calculated from a root density survey. Equation 8 and Equation 9 are nearly equivalent, deviating only slightly in the upper 20% of the rooting zone.

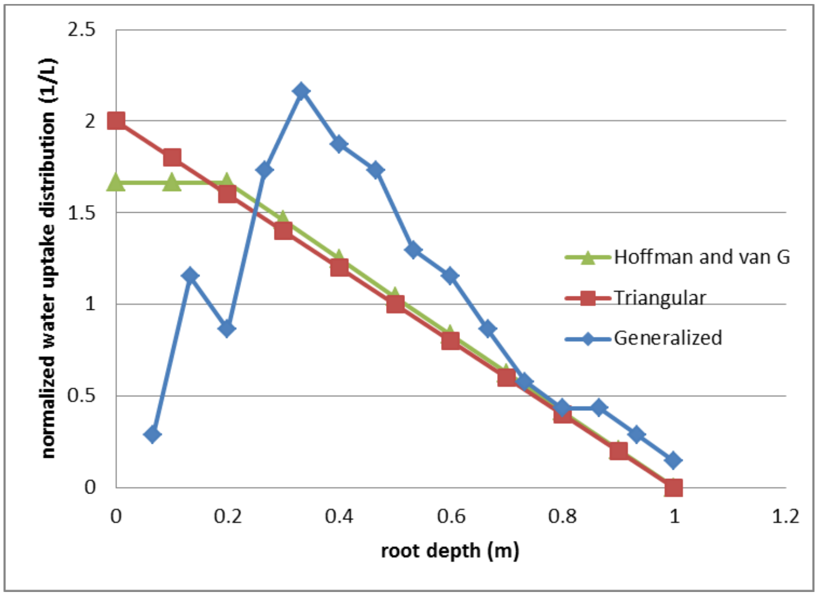

Figure 2. Normalized water uptake distribution.

Defining the Root Water Uptake Boundary Condition

The RWU boundary condition is defined as part of the LCI boundary condition. The required inputs are defined by generalized spline functions that are contained within the Vegetation Data. The functions include:

1. Maximum root depth versus time;
2. LAI versus time;
3. SCF versus LAI;
4. Plant moisture limiting factor α_{rw} versus matric suction (e.g. Figure 1);
5. Normalized root density versus normalized root depth.

The normalized water uptake π'_{root} (Equation 5) is mathematically convenient because the percentage of the root uptake can be evaluated directly at any depth. However, the root depth may change over the growing season. In order to allow for an arbitrary root distribution and variability in the maximum root depth, the root density distribution is defined in the SEEP/W using a normalized root density versus normalized root depth function:

$$\frac{\pi_{root}}{\pi_{root}^{max}} vs \frac{r}{r_{max}}$$ Equation 10

That is, the root length density π_{root} at every depth is normalized by the maximum value in the profile $\pi_{root}^{r_{max}}$ and defined versus the normalized depth. SEEP/W performs the integration of the function and other required calculations to obtain the normalized water uptake π'_{root} at any particular depth. Closed-form equations like those shown in Equation 8 and Equation 9 can be calculated in a spreadsheet over some arbitrary r_{max} and normalized as per Equation 10. The resulting functions take the form shown in Figure 3 (compare to Figure 2).

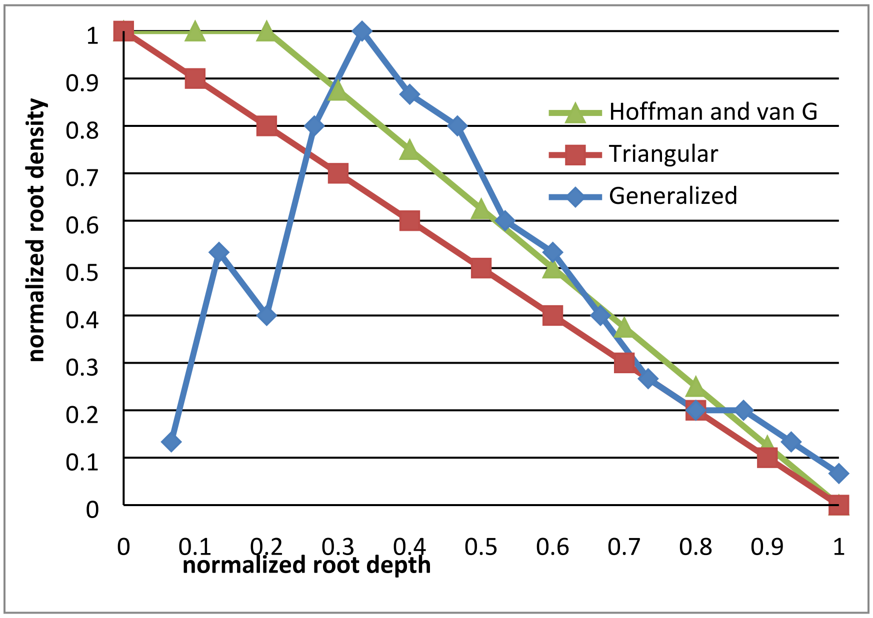

Figure 3. Normalized root density versus normalized root depth.

The LCI boundary condition is applied to the ground surface. SEEP/W automatically identifies and applies the RWU boundary condition (Equation 4) to those elements that exist vertically beneath the LCI (Figure 4). Root depth is always taken as vertical from the ground surface, even if the ground surface is sloping (Figure 4). The LCI boundary condition can be uniquely defined for different parts of the ground surface in order to simulate different types of vegetation. Figure 4 illustrates a uniquely defined boundary condition between points A and B and C and D. The maximum root depth is drawn uniformly; however, each RWU boundary condition can have different vegetation properties, including maximum root depth.

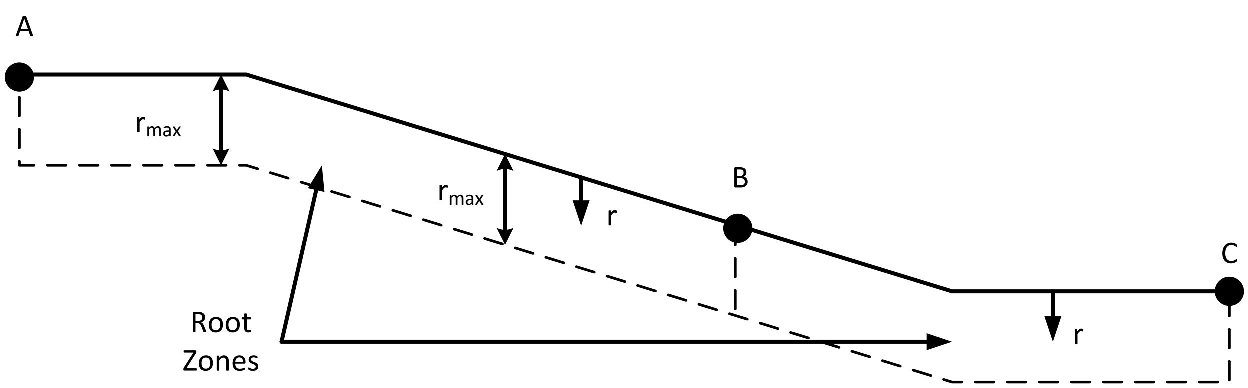

Figure 4. Definition of the root zone beneath the LCI boundary condition

Numerical Experiment

The model domain comprises a 4 m thick stratigraphic unit with one section sloping at 3 to 1 from an x-coordinate of 10 m to 22 m (Figure 5). The material is defined by the saturated-unsaturated material model with a silt volumetric water content function and a constant hydraulic conductivity of 1.0E-09 m/s. The initial conditions for the transient analysis are developed by a steady-state analysis solved with a pore-water pressure of 0 kPa applied to the lower boundary. The transient analysis has duration of 100 days solved with 60 linear steps. A global element size of 0.5 m was used.

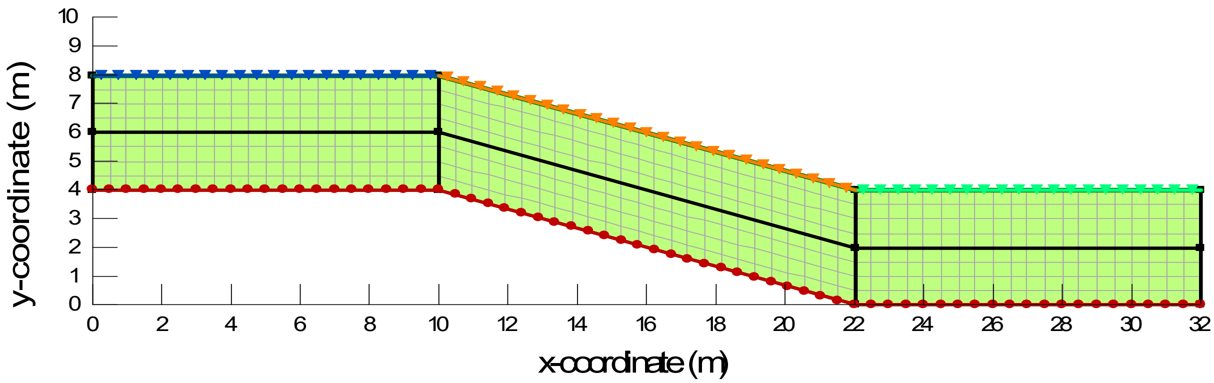

Figure 5. Model domain with three unique sections.

There are three unique LCI boundary conditions applied to the domain for the transient analysis (Figure 5). The User Defined LCI Evapotranspiration Method is used by each boundary condition along with an arbitrarily defined climate data set and a potential evapotranspiration function defined as a constant of 8 mm/day (refer to the Climate Data). The Vegetation Data for each boundary condition uses the following:

1. Root Depth function: the maximum root depth is constant in time at 2 m;
2. Plant Moisture Limit function (refer to Figure 1): the reduction factor α_{rw} increases from 0 to 1 between S1 = 0 kPa and S2 = 5 kPa, remains constant up to S3 = 100 kPa, and then decreases to zero at S4 = 1500 kPa.
3. Leaf Area Index: is constant in time at 5.0;
4. Soil Cover Fraction: is constant at 1.0 for all values of LAI.

The evaporative flux at the ground surface is forced to zero by setting the SCF equal to 1.0 (refer to the Vegetation Data). The uniqueness of the boundary conditions is found in the definition of the root density function definitions of the Vegetation Data sets. The upper, middle, and lower portions of the ground surface are using the Prasad (1998), measured, and Hoffman and van Genuchten (1983) normalized root density functions, respectively, shown in Figure 3.

Results and Discussion

Figure 6 presents the cumulative water-rate time history of all nodes in the root zone beneath each section of the ground surface. The graphs were created by toggling on the option to "Sum (Y) versus Average (X)" for a plot of water rate (Y) versus time (X) at all nodes within each unique root zone. At the onset of the analysis, the matric suction within the root zone is greater than 5 kPa but less than 100 kPa. The actual transpiration flux (Equation 7) is therefore unlimited and equal to the potential evapotranspiration flux via substitution of Equation 4 into Equation 3 with the $\alpha_{rw} = 1.0$ and α_{rs} implicitly equal to 1.0. Multiplication of the potential evapotranspiration flux (8 mm/d) by the surface area of the each slope length yields the actual root water uptake as a volume rate. The 10 m upper and lower sections of the ground surface should therefore have an actual (total) root water uptake of 9.26E-07 m^3/s (i.e. 8 mm/d x 10 m x 1 m) and the mid-slope section 1.17E-06 m^3/s (i.e. 8 mm/d x 12.65 m x 1 m). The values were simulated by SEEP/W.

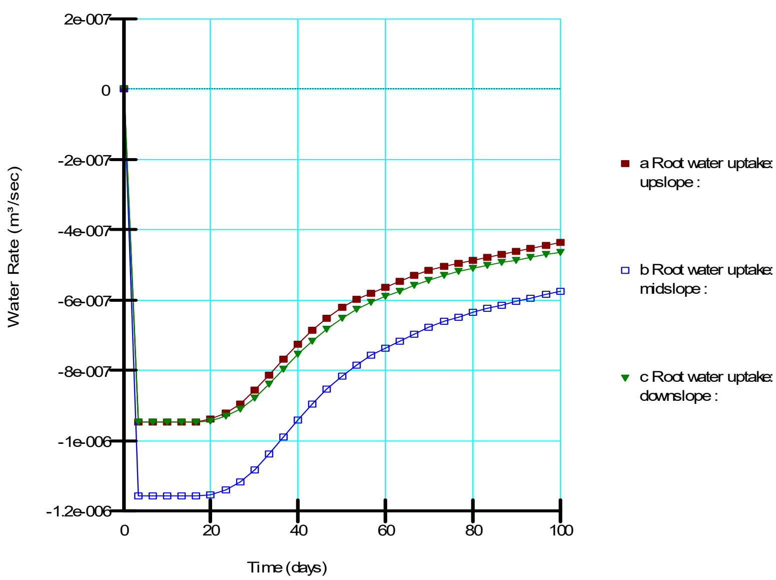

Figure 6. Actual root water uptake.

Figure 7 shows the time history of the pore-water pressure profiles at x-coordinates of 0 m, 15.76 m (mid-slope) and 32 m (Figure 5). Figure 8 shows the pore-water pressure time-history and the node at the top of each profile. The left and right extents of the domain reach a suction of 100 kPa at around Day 13. The mid-slope position reaches this limiting suction about 2 days later. The reduction in the root water uptake is evident in Figure 6. Wilting never occurs because the matric suction remains below 1500 kPa at all points within the root zone.

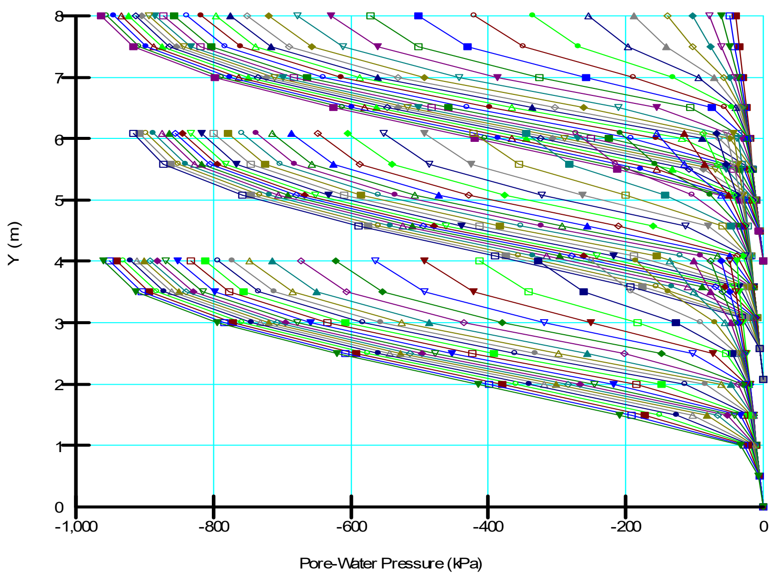

Figure 7. Pore-water pressure profile time histories: left side of the domain (elevation 4 to 8 m), mid-slope, and right side of the domain (elevation 0 to 4 m).

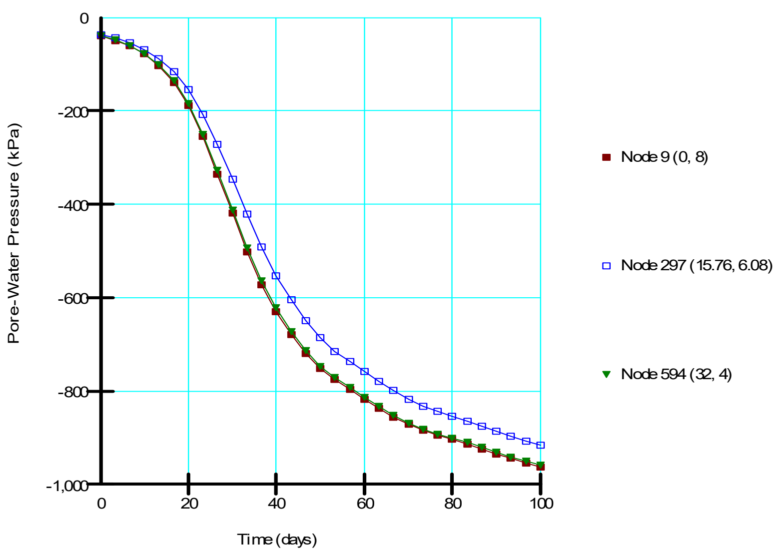

Figure 8. Pore-water pressure-time history and the top of each profile shown in Figure 7.

The Prasad (1988) and Hoffman and van Genuchten (1983) distributions both assumed the greatest potential root water extraction to occur in the shallower soil horizons. Such an assumption is reasonable in temperate climates where precipitation constantly recharges the near surface, but may not be valid in drier climates where seasonal desiccation increases the matric suction past the wilting point. Drying in the upper soil horizon can drive the roots deeper, which was loosely mimicked by the measured function (Figure 3). Variability in the actual root water uptake was minimal between the 3 sections of the soil profile despite the difference in the root density distributions because of the assumptions regarding the material properties and boundary conditions.

Commentary on Mesh Density

The RWU boundary condition (Equation 4) is evaluated uniquely at each gauss point within the root zone. As such, it is best practice to ensure that at least one element exists within the root zone at the point in time when the root depth is at a minimum. Furthermore, the mesh density should be increased in accordance with the variability in the root density with depth.

Figure 9 shows the root water uptake rates simulated using a global element size of 2 m versus 0.5 m used to generate Figure 6. The simulated root water uptake was reasonable despite having only 1 element, and 4 Gauss points, within the root zone. Regardless, the mesh density should be increased given the root distribution and maximum root depth.

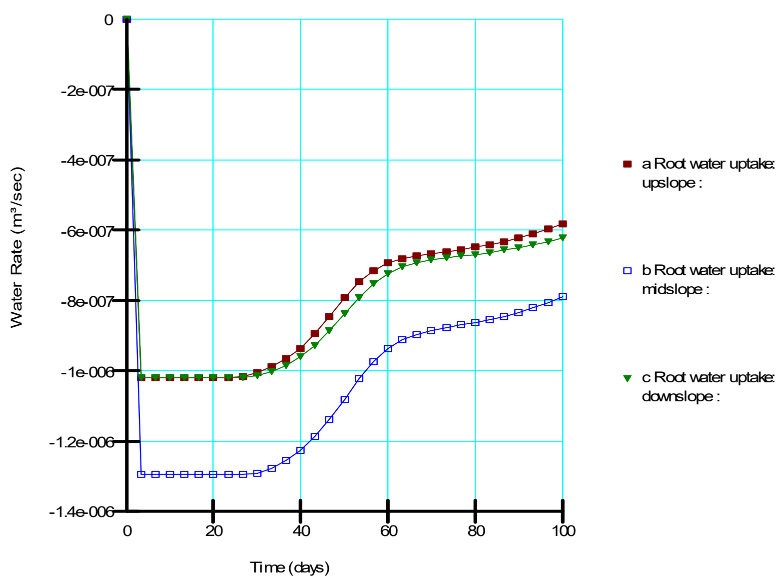

Figure 9. Actual root water uptake simulated using a global element size of 2 m.

Summary and Conclusions

Transpiration by root water uptake is one component of the LCI boundary condition. The other component is evaporation. This example analyzed root-water uptake in the absence of evaporation by defining the SCF as 1.0 at any LAI.

The LCI boundary condition can be uniquely defined along the ground surface to account for variations in the vegetation. Arbitrary root density distributions can be defined and the maximum root depth can vary during the duration of the analysis. The normalized root density versus normalized root depth function can be thought of as extending and collapsing with time in accordance with the maximum root depth. At all points in time, the software determines the elements beneath the LCI boundary condition and calculates the actual root water extraction rate uniquely at each gauss point within the root zone. The accuracy of the solution is therefore dependent to a certain extent on the mesh density.