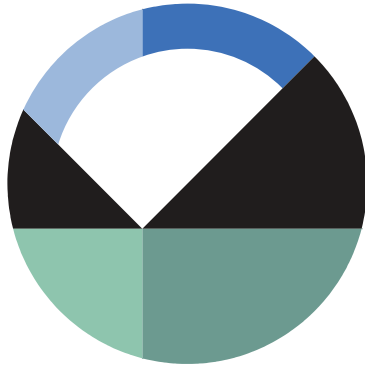


# Progressive Failure of a Cut in London Clay due to Strain Softening

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## Introduction

Potts et al. (1997) used a finite element analysis to explore a link between strain softening and delayed failures of old railway slopes cut into stiff plastic clays. Strain softening causes a soil's strength to reduce from a peak to residual state. The strength loss does not occur simultaneously along a potential rupture zone because the rate of softening is linked with the magnitude of shear straining. As a result, the failure is progressive in the sense that the rupture surface propagates through the soil profile over time. Potts et al. (1997) simulated the progressive failure by using an elastic-plastic model in which strain softening was accommodated by allowing the effective strength properties to vary with deviatoric plastic strain.

The primary objective of this example is to demonstrate how SIGMA/W's coupled formulation can be used to simulate the excavation process and associated time-dependent pore-water pressure response and deformations (i.e. swelling). The excavation and swelling analyses are representative of a non-strain softening soil. A secondary objective of this example file is to demonstrate how the strength reduction technique implemented within SIGMA/W can be used to capture the essence of a progressive failure. SIGMA/W does not have a strain-softening model; consequently, a progressive failure cannot be modeled in the truest sense. Specifically, the strength reduction technique cannot be used to simulate the time-dependency of a progressive failure, which occurs simultaneously – or as a result of – the swelling phase. Rather, the strength reduction technique is applied after the swelling phase to determine the possibility of a progressive failure and explore the location and shape of the critical slip surface.

## Background

A rapid excavation into low hydraulic conductivity clay unloads the soil and causes a tendency for volumetric expansion. The tendency for volumetric expansion is offset by a reduction in the pore-water pressures, resulting in a nearly un-drained response. Over time the soil affected by excavation imbibes water and swells, causing a reduction in the mean effective stresses. The reduction in mean effective stress in some areas of the soil domain can bring the stress states onto the failure surface. The overall collapse of the slope is therefore delayed by the time required for pore-water pressure equilibration.

Progressive failure refers to the non-uniform mobilization of shear strength along a potential rupture surface. In the case of a cut slope, the area near the toe at the base of the excavation may have mobilized the peak shear strength by the end of construction. The rupture surface will propagate further into the slope once the excavation is complete and the pore-water pressures begin to recover (mean effective stresses decrease); that is, more of the domain will reach peak strength. At the same time, the already failed soil experiences an ever increasing amount of shear strain that can cause a loss of strength. This mechanism is referred to as strain-softening. At collapse, part of the rupture surface will have a post-peak shear and perhaps approach a residual strength, while part of the rupture surface will not even have formed (Potts et al., 1997). Therefore, the average strength of the soil along the rupture surface at collapse must be less than the peak strength, but greater than the residual strength.

## Numerical Simulation

The SIGMA/W coupled stress-strain and water transfer formulation was used to simulate the excavation of a slope cut into a (non-strain softening) stiff plastic clay. A coupled analysis was also used to simulate the equilibration of the pore-water pressures and resulting deformations that occurred once the excavation was complete (i.e. swelling). As noted previously, SIGMA/W does not have a strain-softening model; consequently, the progressive failure was explored post-swelling using the strength reduction technique.

The behavior of the strain softening Brown London Clay was simulated using the elastic-(perfectly) plastic Mohr-Coulomb model. The soil properties used to represent the stress-strain behavior and strength of the clay are shown in Table 1. Young's modulus was varied with vertical effective stress in much the same manner as Potts et al. (1997). The initial stresses were established with an earth pressure coefficient of 1.5.

SIGMA/W's fully coupled consolidation analysis requires specification of the volumetric water content (VWC) and hydraulic conductivity functions. The VWC sample function for clay was used in the analyses. The hydraulic conductivity was assumed constant throughout the domain. Some of these material properties are simplifications of the parameters used by Potts et al. (1997).

The entire analysis tree and finite element domain and boundary conditions are shown in Figure 1 and Figure 2, respectively. The initial *in situ* stresses were established assuming hydrostatic pore-

water pressure conditions with the water table 1 m below ground surface. The excavation was 10 m deep with 3:1 side slopes. The excavation phase was simulated by deactivating 4 regions of 1 m thickness and three regions of 2 m thickness. Each excavation phase was simulated using one time step of duration 10 days, resulting in the excavation being completed in 70 days.

Table 1. Soil properties for the soft clay.

Parameter	Value
Young's modulus (kPa)	Variable (min 4000 kPa)
Poisson's ratio ( $\nu$ ):	0.2
Effective friction angle	20
Effective cohesion (kPa)	7
Unit Weight ( $\gamma$ ; kN / m <sup>3</sup> ):	18.8
K <sub>sat</sub> (m/day)	4.3 × 10 <sup>-5</sup>
Earth Pressure Coefficient	1.5



Figure 1. Analysis tree for the Project.

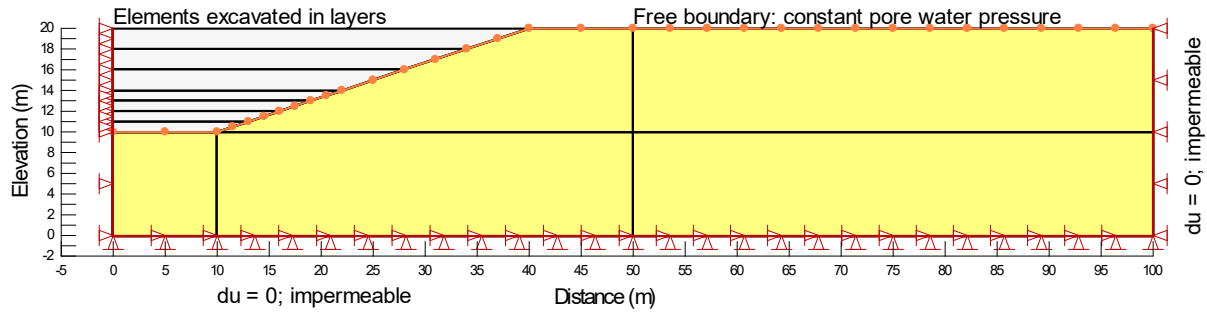


Figure 2. Finite element mesh and boundary conditions.

The swelling phase was then simulated for approximately 30 years (or 11,000 days). During the swelling phase, the pore-water pressure was maintained at -10 kPa along the surface boundary. The upper hydraulic boundary condition represents the average measured surface pore-water pressure (Potts et al, 1997).

The progressive failure was simulated post-swelling using the strength reduction technique (Griffiths and Lane, 1999). The approach differs from that used by Potts et al. (1997). The method involves reducing the strength in increments, computing a load imbalance, and applying the load imbalance as a boundary condition. The factored strength parameters are given by:

$$c_f' = \frac{c'}{FOS} \quad \text{Equation 1}$$

and

$$\phi_f' = \text{atan} \left( \frac{\tan \phi'}{FOS} \right) \quad \text{Equation 2}$$

The strength was reduced in three stages and, therefore, three strength reduction analyses were completed (Table 2).

Table 2. Soil properties for the soft clay.

SRF	Phi'	c'
1	20	7
1.1	18.3	6.4
1.15	17.6	6.1
1.2	16.9	5.8

The slope stability analyses were conducted using SIGMA/W stresses for each time step of the swelling analysis and after each strength reduction analysis. A SIGMA/W Stress slope stability analysis uses the effective stress information from the stress-strain analysis to compute the base effective normal stress and then compute the shear strength. A localized factor of safety can then

be computed by comparing the strength with the simulated shear mobilized along the rupture surface. The stability analyses also served as means to judge complete collapse.

### Results and Discussion

The extent of the piezometric line depression as a result of the unloading is rather dramatic (Figure 3). A total of 10 m of soil was removed, causing a decrement in the pore-water pressure just beneath the base of the excavation of over 120 kPa (Figure 4). The pore-water pressures are still recovering after 8000 days (about 22 years). The yield (failure) zone is confined near the corner of the base of the excavation at the end of construction (Figure 5). The swelling phase causes a reduction in the mean effective stress, causing the yield (failure) zone to propagate deeper into the slope. Most of the soil beneath the base of the excavation is failed by the end of the swelling phase.

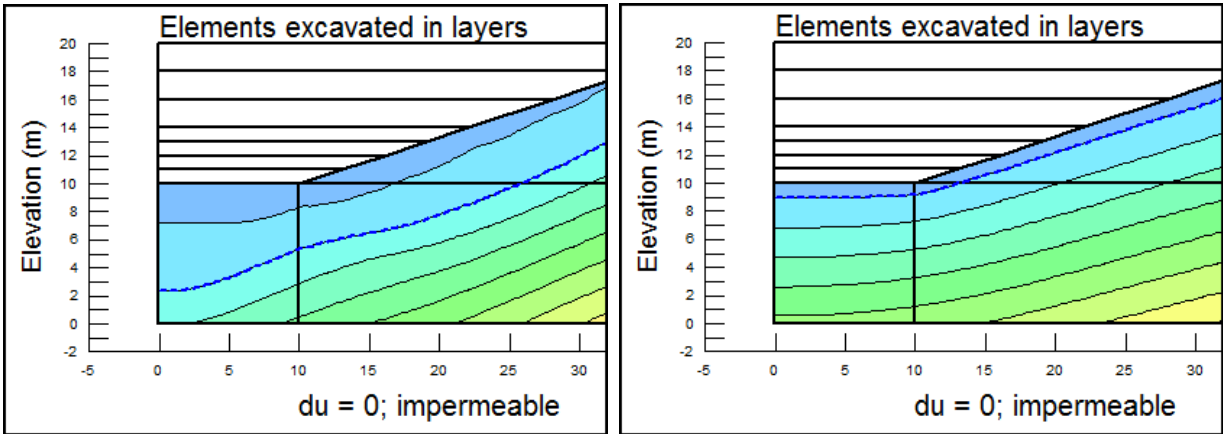


Figure 3. Piezometric lines at the end of construction (left) and at the end of the swelling phase (right).

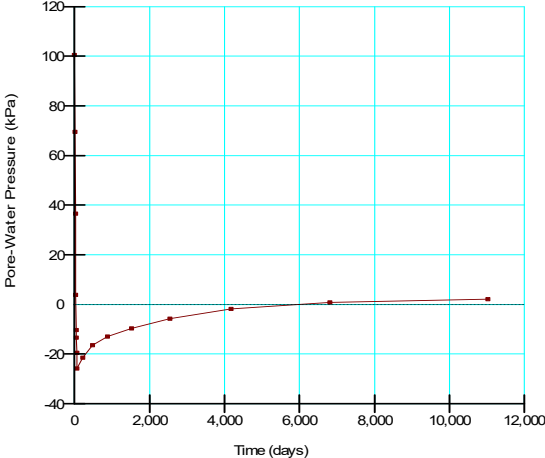


Figure 4. Pore water pressure response at Elevation 8.75 m along the centerline of the excavation.

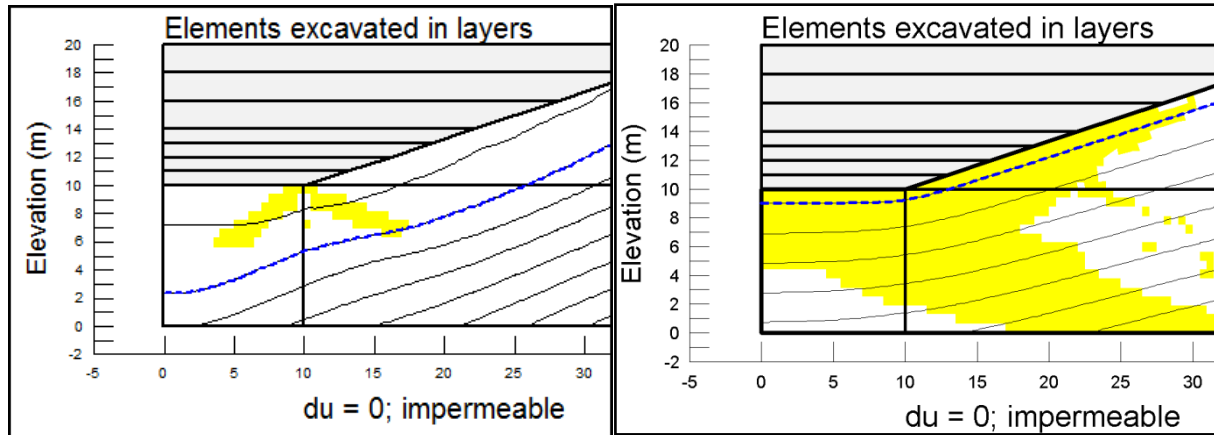


Figure 5. Extent of yield (failure) zone at the end of construction (left) and at the end of the swelling phase (right).

Figure 6 presents a factor of safety versus time plot for the swelling phase. The analysis, which is using stresses and pore-water pressures from the SIGMA/W analyses, indicate a gradual decrease in the factor of safety during the swelling phase. This time-dependent reduction of the factor of safety is in-keeping with the propagation of the failure zone into the slope. The long-term factor of safety at the end of the analysis was around 1.3. The excavation and swelling analyses are representative of a non-strain softening soil.

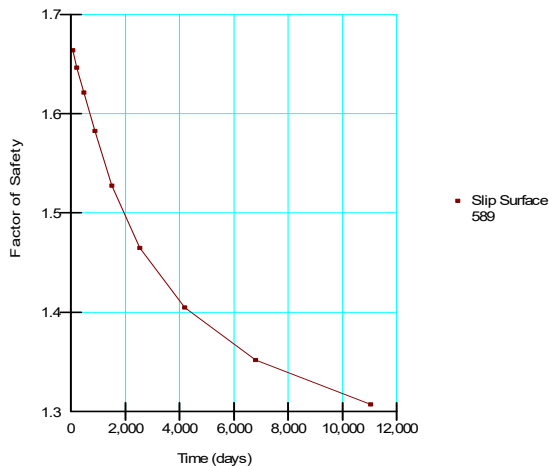


Figure 6. Factor of Safety versus time during the swelling phase.

According to the progressive failure mechanism theory, the strength for soils that have been heavily sheared is between the peak and residual values. In contrast to the strain-softening method used by Potts et al. (1997), the strength reduction technique used by SIGMA/W reduces the strength of all elements in the entire domain at the end of the swelling phase. There is no consideration given to the magnitude of accumulated shear strains and associated reduction of strength during the swelling phase. As a consequence, the strength reduction method cannot capture a progressive failure in the truest sense because the strength is not permitted to vary along the rupture zone in accordance with shear strains that are accumulating during the swelling phase. Furthermore, examination of the yield

zone generated by the strength reduction is not particularly useful because the yield zone spreads out considerably more than would be predicted by using a strain-softening model.

Regardless of the aforementioned deficiencies, the strength reduction technique does, in essence, ‘target’ elements with high shear strains. This occurs because these elements also have high deviatoric stresses, which means that a reduction in the shear strength is going to create a large load imbalance in specific areas of the domain. Figure 7 presents contours of deviatoric strain after the second and third strength reduction analyses and Figure 8 shows the corresponding factors of safety. A rupture zone is clearly propagating towards surface after the second strength reduction and collapse occurs (i.e. the factor of safety = 1.0), with part of the rupture surface not yet formed. The reduced strength properties of 17.6 degrees and 6.1 kPa would represent an average strength through both the developed and undeveloped rupture zone.

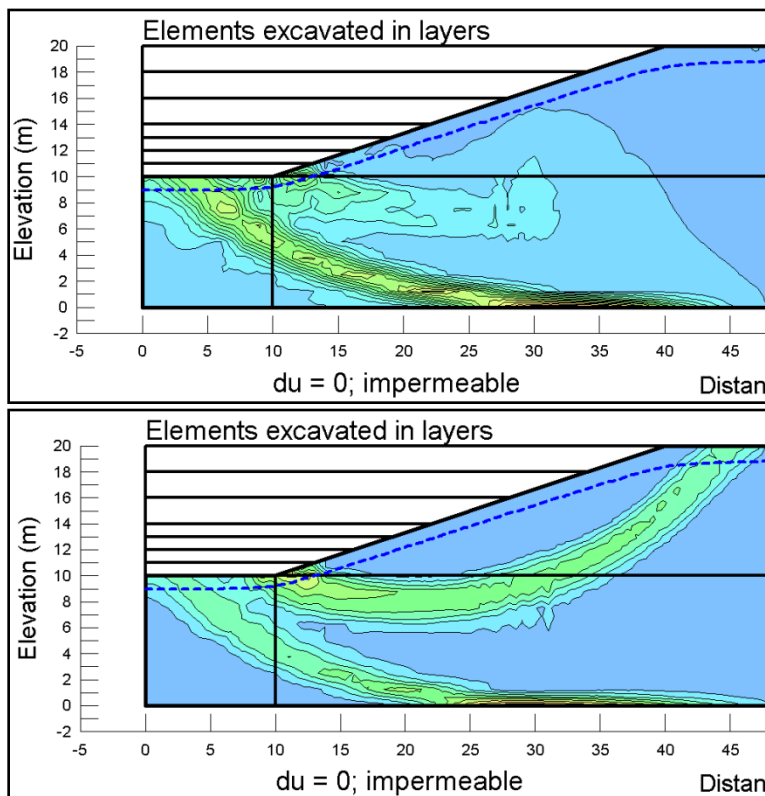


Figure 7. Deviatoric strain contours after the second (top) and third (bottom) strength reduction analyses.

The third strength reduction analysis attempts to load the soil beyond a state of limiting equilibrium. As a result, the rupture surface – clearly indicated by the deviatoric strain contours – propagates to the ground surface and the factor of safety decreases below 1.0 (Figure 7 and Figure 8). It is worth noting that the analysis converged, although with some difficulty, which suggests that the use of non-convergence alone as a judge of failure is not well suited for finite element stability analyses (note: Potts et al. (1997) considered both non-convergence and residual stresses).

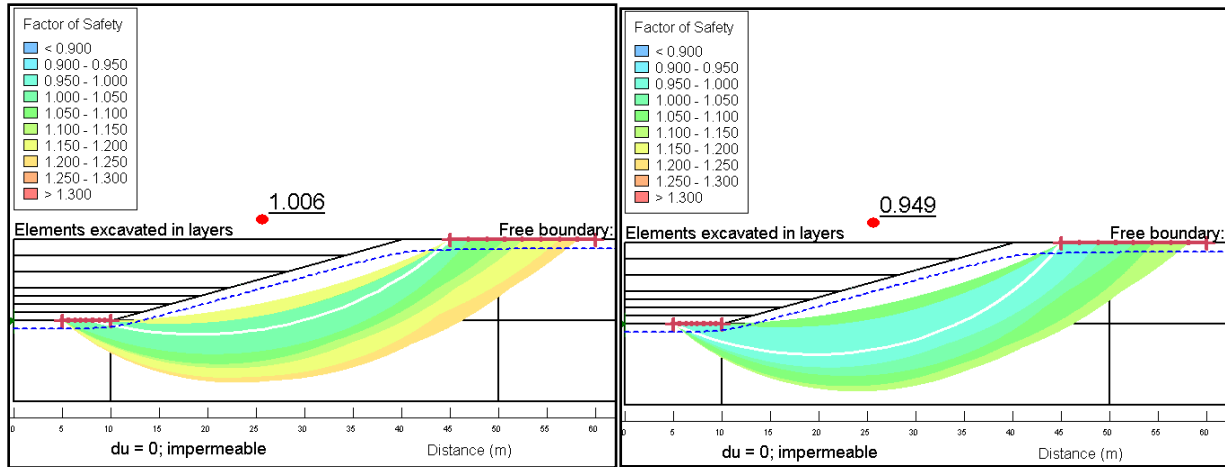


Figure 8. Factor of safety after the second (left) and third (right) strength reduction analyses.

Figure 9 presents the relative displacements resulting from the strength reduction technique (i.e. post swelling analysis). The absolute values of the displacements are of no significance other than to note that the largest displacements are constrained within the rupture zone. One of the proposed advantages of finite element stability analyses, over a limit equilibrium analysis, is that there are no kinematic constraints placed on the shape of the slip surface. In this case, however, the circular slip surface imposed by SLOPE/W adequately captured the shape of the slip surface predicted by SIGMA/W.

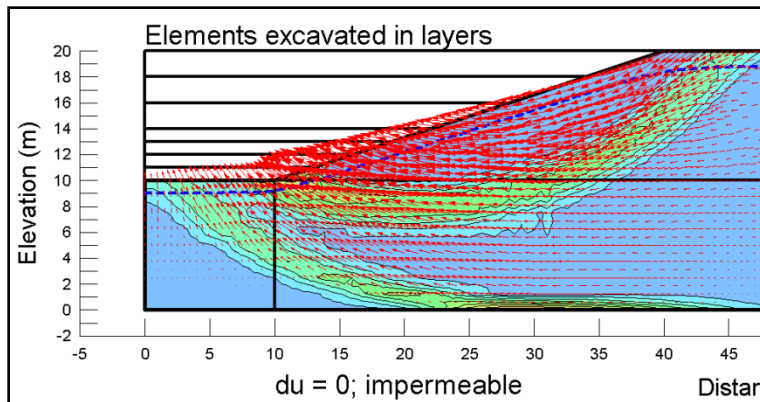


Figure 9. Displacement vectors after the third strength reduction analysis.

As noted previously, Potts et al. (1997) used a strain-softening model to simulate the progressive failure mechanism because the approach allows the strength to vary in accordance with the deviatoric strain. The shape and location of the critical slip surface is highly dependent on strength definition of the soil, and, in this case, the spatial variation of the strength. As such, the strain-softening approach of Potts et al. (1997) generated a differently shaped slip surface (Figure 10). It is also interesting to note that the strength varied from the residual values (13 degrees and 2 kPa) along the base of the rupture zone to the peak values (20 degrees and 13 kPa) along the back-scarp



(Figure 10). As noted previously, the strength reduction technique required an average reduced strength of 17.6 degrees and 6.1 kPa along the entire slip surface.

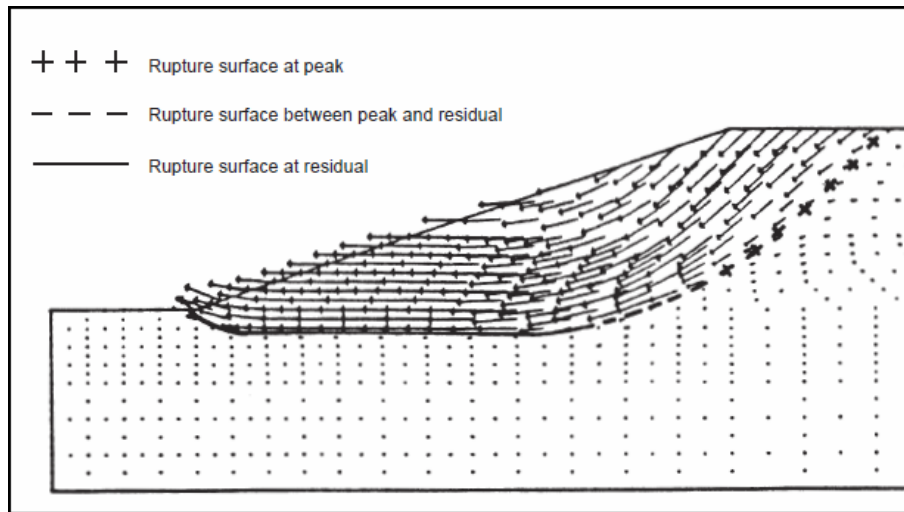


Figure 10. Strain softening failure surface from Potts et al. (1997).

## Summary and Conclusions

The coupled formulation can be used to simulate the pore-water pressure response and deformations that occur during and after excavation in a non-strain softening soil. The strength reduction technique implemented in SIGMA/W can be used subsequently to capture the essence of a progressive failure mechanism; however, there are some limitations. The most apparent of these limitations is the loss of information regarding the time to collapse. The progressive failure (i.e. strain softening) actually occurs simultaneously and as a result of the pore-water pressure equilibration that occurs post-excavation. The strength reduction technique, however, completed at the end of swelling (which was modeled using peak strengths) and, therefore, cannot be used to judge the time to failure. From a practical perspective, however, the approach can be used to effectively investigate the possibility of a progressive failure and its associated mechanism for failure.

## References

- Griffiths, D.V. and Lane, P.A. (1999). Slope stability analysis by finite elements, *Géotechnique*, 49(3), 387-403.
- Potts, D.M., Kovacevic, N., Vaughan, P.R. Delayed collapse of cut slopes in stiff clay. *Géotechnique* 47, No. 5, 953-982.