Introduction

The International Conference on Geotechnical and Geological Engineering was held in Melbourne, Australia in November 2000. The conference is referred to here as GeoEng2000.

At the conference, a series of invited keynote lectures were presented. One of the keynote lectures was titled, *Computing and Computer Modelling in Geotechnical Engineering*. This keynote paper was prepared by J.P. Carter, C.S. Desai, D.M. Potts. H.F. Schweiger and S.W. Sloan. Highlights of the paper were presented at the conference by John Carter, Challis Professor, Department of Civil Engineering, University of Sydney, Sydney, NSW, Australia. (a printed version of the paper is in the conference proceedings, Volume 1 (Keynote Papers), pages 1157 to 1252).

One of the topics of this keynote paper was *Validation and Calibration of Computer Simulations*. This section describes some of the work by the German Society for Geotechnics, which has worked on establishing some benchmark problems for validating numerical analyses. One of the benchmark examples is about the construction of a tie-back wall for a deep excavation in Berlin.

Part of the numerical validation program included holding a typical analysis competition. Information about the project was made available to those wishing to model the construction and make a prediction of the wall performance. The lateral wall deflection was measured with an inclinometer and the main objective was to see if the analyst could predict the lateral wall deflection. The measured results were not made available to the analyst until the end of the competition. A total of 15 organizations (universities and consulting companies) participated.
The objective here is to demonstrate that SIGMA/W can be used to do this type of analysis. More specifically, the objectives are to:

- Demonstrate that SIGMA/W has sufficient features and capabilities to simulate the construction sequence;
- Illustrate the procedures and techniques that are required to obtain close agreement between the predicted and measured wall deflections; and
- Highlight the key modeling issues in a situation like a tie-back wall with pre-stressed anchors.

**Background**

Figure 1 shows a schematic diagram of the tie-back wall. The natural site conditions consist of sand throughout, with the water table 3 m below the ground surface. Basically, the design involves constructing a diaphragm wall, excavating down 16.8 m in four stages and tying the wall back with three rows of pre-stressed anchors.

![Figure 1. Schematic of the Berlin tie-back wall.](image)

When analyzing a field problem like this, it is usually adequate to round-off the dimensions. For this analysis, a decision was made to work to the nearest metre. In the context of the accuracy with which the material properties can be defined, there is no value in refining the dimensions to the nearest tenth of a metre, for example. Moreover, it is highly unlikely that the contractor can make an excavation exactly to 14.35 m below the ground surface.

Good modeling practice dictates that the problem should not be unnecessarily complicated.
The properties of the diaphragm wall were specified as:

- $E = 30,000$ MPa $= 30,000,000$ kPa $= 3 \times 10^7$ kPa (typical of concrete)
- Poisson's ratio ($\nu$) $= 0.15$
- Unit weight ($\gamma$) $= 24$ kN/m$^3$

From a SIGMA/W analysis perspective, it is the flexural (bending) stiffness of the wall that is important. This stiffness is best included as beam elements.

The parameters for the beam elements are:

- $E = 3 \times 10^7$ kPa
- Cross sectional area $= 0.8 \times 1.0 = 0.8$ m$^2$ (thickness of wall is 0.8 m)
- $I$ (moment of inertia) $= bh^3 / 12 = 1.0 \times 0.8^3 / 12 = 4.3 \times 10^{-2}$ m$^4$

The difference between the wall and soil unit weight is ignored in the analysis. Poisson’s ratio is also not required for a beam type element in SIGMA/W.

The anchors are steel bars about 43.7 mm (1.75 inches) in diameter. The cross sectional area was specified as 15 cm$^2$. This equals 1500 mm$^2$ or $1.5 \times 10^{-3}$ m$^2$ (all length units in SIGMA/W must be the same; in this case metres).

$E = 2.1 \times 10^8$ kPa $= 210 \times 10^6$ kPa $= 210$ GPa (similar to structural steel).

The horizontal spacing of the anchors along the wall is shown on the diagram in Figure 1.

For a 2-D analysis, the actual pre-stress forces must be specified per unit width of wall. The pre-stress anchor forces for the SIGMA/W analysis consequently are:

- Row 1: 334 kN
- Row 2: 700 kN
- Row 3: 726 kN

The force in a bar relative to the strain is,

$$F = EA \frac{\Delta L}{L}$$  \hspace{1cm} \text{Equation 1}

where $E$ is the stiffness modulus, $A$ is the cross-sectional area, and $(\Delta L/L)$ is the strain. If the anchor force ($F$) is to be normalized per unit length of wall (1 unit into the page), then the right side of the equation also needs to be normalized per unit length of wall. Both sides of the equation need to be divided by the anchor spacing. Either $E$ or $A$ on the right side can be divided by the spacing. In this example, the cross-sectional area $A$ is divided by the spacing.
The soil at this Berlin site is medium dense sand with the following specified properties:

- Ø' = 35 degrees
- γ = 19 kN/m$^3$
- $K_o = 1 - \sin \theta = 1 - \sin 35 = 0.43$

The submerged unit weight was specified as 10 kN/m$^3$. If $\gamma_w$ is taken as 10 kN/m$^3$, then the saturated (below water table) unit weight is 20 kN/m$^3$. The difference between the above and below water table unit weights (if indeed there is any) was ignored in this analysis. A total unit weight of 20 kN/m$^3$ was used throughout for the in situ conditions. Any small variations in unit weight are of little consequence in this analysis and approximate values are more than adequate.

The soil stiffness is a critical parameter in this analysis. It is the most difficult parameter to characterize and yet it has the greatest influence on the results. Some data was presented to the competition participants, but the analysts were free to use and judge any data and information available to them, including other published results and experiences from other similar projects.

The suggested Young’s modulus for the sand was:

- $E = 20,000 \times z^{0.5}$ (square root) for the first 20 m below the ground surface, and
- $E = 60,000 \times z^{0.5}$ below the top 20 m.

This represents an increase in stiffness with depth, as illustrated in Figure 2. The sudden jump in stiffness at the 20 m level seems unrealistic. This is likely not representative of actual conditions. A more gradual transition is more likely.

![Figure 2. Specified variation of soil stiffness (E) with depth.](image-url)
While some information was provided on the soil stiffness properties, it was the intention that the competition participants would exercise their judgment as to appropriate values. The competition criteria did not require that the analysts use the values suggested.

With the above as a guide, the E-modulus function adopted for this analysis is shown in Figure 3. The distribution is a function of the overburden. The minimum E-modulus near the ground surface is 35,000 kPa, and then increases with depth (overburden) to about 20 m (El 40 m) below the ground surface. Below that, the E-modulus transitions to the stiffer sand at depth. The maximum value at the base of the problem is 465,000 kPa.

Figure 3. Soil stiffness (E) as a function of depth (elevation) below the ground surface.

The purpose of the hydraulic barrier is not clear from the information provided. The likely purpose is to prevent water from flowing up into the excavation. If this is true, then it can be assumed that the ground behind the wall remains saturated and the water table elevation does not change. The water table is only lowered inside the excavation. This is a key assumption when it comes to determining the pressure on the wall.

In this analysis, the hydraulic barrier is assumed to have a constant E-modulus of 100,000 kPa.

One of the key issues in an analysis like this is the initial insitu stress state. The wall performance is strongly related to the pressures the wall needs to retain, and this is directly related to the stresses in the ground before construction starts.

The first step therefore is to do an Insitu-type of analysis.

In SIGMA/W the earth pressure at rest $K_o$ is control through Poisson’s ratio $\nu$. Recall that for a 2-D plane-strain analysis:

$$K_0 = \frac{\nu}{(1 - \nu)}$$

Equation 2
$K_0$ can be estimated from:

$$K_0 = 1 - \sin \phi'$$  \hspace{1cm} \text{Equation 3}

For $\phi' = 35$ degrees $K_0 = 0.43$ and for $K_0$ equal to 0.43, the equivalent $v$ is equal to 0.3.

The total unit weight is 20 kN/m$^3$, and the water table is 3 m below the ground surface. The unit weight of water is rounded-off to 10 kN/m$^3$.

Figure 4 shows the horizontal ($X$) total and effective stresses along a vertical profile.

The effective horizontal stress at the bottom of the profile should be approximately:

$$\left(20 \text{ kN/m}^3 \times 60 - 10 \text{ kN/m}^3 \times 57\right) \times 0.43 = 271 \text{ kPa}$$  \hspace{1cm} \text{Equation 4}

The total horizontal stress should be:

$$271 + \left(10 \text{ kN/m}^3 \times 57\right) = 841 \text{ kPa}$$  \hspace{1cm} \text{Equation 5}

Both match the values at the bottom of the graphs in Figure 4.

Figure 4. Total and effective horizontal stress profiles.

The effective stress profile near the ground surface curves to the right. This is due to the negative pore-water pressures (suction) above the water table. At the bottom, the effective stress profile bends back (stress becomes less). This is due to the use of 4-noded quadrilateral elements and constant stress and pore-water pressure in the elements. This edge-effect is inherent in 4-noded elements. It does not have any significant effect on the numerical results.
During the excavation, it will be assumed for analysis purposes that the dewatering will be such that the water table is always at the excavation level. In other words, the excavation process removes both the soil and water at the same time.

The excavation process is simulated in a finite element analysis by applying forces on the excavation face equal to, but in the opposite direction of, the forces present before removing the soil. It is the total stress that goes to zero on the excavation face. Stated another way, it is the total stress that acts behind the wall after excavation. By removing the total stress, the excavation simulation accounts for both the soil pressure and the water pressure.

The competition participants were asked to simulate the dewatering down to 17.9 m before removing any soil, even though this was not the procedure used during the actual construction. This separate dewatering step is not included in this analysis here. It is not trivial to simulate dewatering from a stress change point of view and the effect is relatively small on the overall lateral movement of the wall. The effort involved is not warranted in this case. Furthermore, considering the dewatering to take place as excavation proceeds is closer to what actually happens during the construction.

The reason for asking the competition participants to do this initial dewatering step is not clear considering its minor effect on the wall lateral movement.

The total pressure acting along the top 17 m of the wall prior to making the excavation is shown in Figure 5. Once the soil has been removed on the left, the shoring system will be subject to this pressure.

Figure 5. The total lateral pressure profile at the wall location when the water table is included.

Figure 6 shows the same profile if the water table is ignored in the in situ analysis. Note that the pressure is substantially less.
Figure 6. The total lateral pressure profile at the wall location when the water table is not included.

The area under the curves in the above two figures is an approximation of the total lateral force that will act on the shoring system. When the water table is included, the total force is about 1870 kN per metre of wall (into the page); without the water table in the in situ analysis, the total force is around 1230 kN. This illustrates how the effect of the groundwater comes into the analysis, and how the pore-water pressure affects the lateral pressure on the wall. Viewing these pressure diagrams provides a good reference picture for later interpreting and judging the results.

It is of interest that the sum of the anchor pre-stress forces is close to the total wall force represented by the above pressure diagram. The sum of the anchor pre-stress forces per metre of wall is 1760 kN; the total force represented by the wall pressure is 1870 kN, as noted earlier. This being the case, we should expect relative small wall displacements. (It would be interesting know whether the designers gave this consideration when the anchor system was established).

The total lateral stress along the profile of the wall as given in Figure 5 follows a hydrostatic distribution. The pressure at the 43-m level is 220 kPa. If we assume that the pressure distribution is linear, then the rate of increase is about 13 kPa per metre with depth. This information can be used in a SIGMA/W boundary condition that represents the removal of the in situ lateral stress acting on the wall. In SIGMA/W, the rate of increase would be specified as -13; the negative sign indicates that the stress is, in essence, pulling on the wall.

Figure 7 shows the best predictions presented by the competition participants. Three other predictions showed much larger lateral deflections – up to 225 mm. These are not included in the figure. Even the 11 best predictions presented in Figure 7 show considerable scatter. Most of the scatter can be attributed to the adopted soil stiffness properties. Of further interest is the fact that many of the competition participants used the same commercially available computer code and used the same constitutive soil model. This further demonstrates that the predictions are very closely tied to the specified soil properties and not so much to the software and associated constitutive model.
Superimposed on Figure 7 is the actual measured wall deflection. At the top of the wall, the deflection is about 10 mm. The maximum deflection is just over 20 mm and occurs at about mid-height of the wall, 10 m below the ground surface.

It is interesting to note that, in the measured deflection profile, the value is zero at the bottom of the wall. The reason for this is unknown. Did the wall base actually not move, or was the inclinometer data not corrected for possible movement at the bottom? Most of the computed profiles in Figure 7 show some displacement at the base, which is what often happens in cases like this. For the measured profile to show zero displacement at the base makes one think that the inclinometer data was not corrected for base movement. If the inclinometer data was corrected for base movement, then the measured displacement profile may be shifted to the left slightly and fall more in the middle of the computed results in Figure 7.

Of more significance than the magnitude of the displacement is the shape of the deflection profile. Most of the competition deflection profiles have a shape not unlike that of the measured profile.
Numerical Simulation

Figure 8 shows the problem configuration used in the SIGMA/W analysis. The details can be viewed and studied by opening the related data file.

Figure 8. The Berlin tie-back wall configuration.

Each analysis is given a duration of one day. Although time does not come into the actual analysis, the time is specified to give the steps some order and to plot results across all the analyses. There are a total of 11 analyses in the Analysis Tree to represent each step in the excavation process (Figure 9).
Analyses

- 1: Initial in situ
- 2: Excavate to 57 m
  - Excavate 3 m
- 3: Excavate to 55 m
  - Excavate 2 m
- 4: Install anchors
  - Install & prestress upper anchors
- 5: Excavate to 53 m
  - Excavate 3 m
- 6: Excavate to 50 m
  - Excavate 2 m
- 7: Install middle anchor
- 8: Excavate to 47 m
- 9: Excavate to 45 m
- 10: Install lower anchor
- 11: Excavate to 43 m

Figure 9. Analysis Tree for the Project.

Interface elements are used on either side of the beam representing the diaphragm wall, as illustrated in Figure 10. These elements make it possible to allow for some slippage between the wall and the soil. In this case the interface material is treated as an elastic-plastic material with a reduced strength.

![Diagram of excavation process]

Figure 10. Illustration of the interface elements on either side of the wall (beam).

Note that the interface is treated as a relatively thin slip zone, as opposed to a very thin slip surface. This is better practically and numerically. Moreover, in reality, it is unlikely that a paper-thin slip surface would actually form in the field – a thin slip zone is much more likely. The strength properties used are $c=0$ kPa and $\phi=35$ degrees.

The unloading at the base of the excavation can be simulated with a y-pressure boundary condition. One y-stress boundary condition represents the removal of 3 m, and the other represents a removal of 2 m.
The application of the excavation boundary conditions is illustrated in Figure 11 when the soil between Elevation 50 and 47 is removed.

![Figure 11. An illustration of the applied excavation boundary conditions.](image1)

SIGMA/W has two procedures for simulating the removal of soil. One procedure is to use boundary conditions to simulate the excavation forces. The other procedure is to allow SIGMA/W to compute the excavation forces based on the stress state in the ground before the soil is removed.

The alternative of allowing SIGMA/W to compute the excavation forces is used in this case history example.

It is difficult to imagine that there will be slippage between the wall and surrounding soil, considering the movements are so small. Nonetheless, an analysis was done where the interface material was given a lower strength and a lower E-modulus. The phi value was reduced to 25 degrees and the E-modulus was set at a constant 10,000 kPa. This has the effect of creating a softer interface material than the surrounding soil.

This results in a slight off-set in the vertical displacement in the lower corner of the excavation, as shown in Figure 12.

![Figure 12. Off-set of the upward movement on either side of the wall when interface elements with a reduced strength are used.](image2)
Results and Discussion

Figure 13 shows the wall deflections for the first two excavation stages and the installation of the upper anchor. Note how the wall moves out (to the left into the excavation) as would be expected, but then is pulled back with the pre-stressing of the upper anchor.

Figure 13. Wall deflections for the first two excavation stages and the installation of the upper anchor.

Figure 14 shows the wall deflections for the last four stages. Note how the largest deflection occurs on Day 8. Pre-stressing the lower anchor pulls the wall back but then moves out again with the removal of the last excavation stage.

Figure 14. Lateral wall deflection during the last four stages.

Figure 15 compares the measured wall deflection with the SIGMA/W computed deflections upon completion of the excavation. The maximum lateral deflections are within about 5 mm of each other.
The SIGMA/W computed deflection at the ground surface is 10 mm, which matches the measured deflection. At the base of the excavation the SIGMA/W deflection is somewhat higher than the indicated measurement; the difference is only about 7 mm.

All factors considered, the SIGMA/W deflection profile is remarkably close to the actual measured profile, particularly the shape of the profile. In the context of all the parameters involved and the accuracy with which the soil properties can be characterized, the computed and measure profiles are, for all practical purposes, identical.

Figure 15. Final measured and SIGMA/W-computed deflection profiles.

Figure 16 shows the SIGMA/W computed deflection profile on the published information discussed earlier. Once again, the SIGMA/W deflections are close to the measured values. Moreover, the SIGMA/W profile shows the best match with the measured deflection profile amongst all the other deflection profiles presented by the competition participants. It could be argued that this is not
surprising, since the measured profile was known prior to doing the SIGMA/W analysis. Still, the SIGMA/W results and profile fall within the range presented by others, and is as good as or better than the others.

Figure 16. SIGMA/W-computed deflections relative to the completion results.

A key component in the design of a retaining wall like this is the maximum bending moments. Figure 17 shows the bending moment variations during the construction of the shoring system. This is an illustration of the type of data available from this type of SIGMA/W analysis.
Of considerable significance is the observation that the maximum moments do not occur when the last material is excavated. The maximum moment occurs on Day 8, not on Day 10 when the last material is removed, as exhibited in Figure 18. This is typical of this type of shoring system.

Figure 19 shows the forces in the free (unbonded) length of the upper anchor. The starting force is -334 kN (negative indicates tension) which is the pre-stress. The force in the bar increases as the next two layers are excavated, but then decreases as the middle anchor is pre-stressed. Then the tension in the bar again increases as the excavation proceeds, and again slightly decreases when the lower anchor is pre-stressed. Finally, the force in the bar is close to the initial design pre-stress force. The middle and lower anchor exhibit similar behavior.
Figure 19. Forces in the upper anchor.

Again the important response demonstrated here is that the force in the bars varies during the construction sequencing and the maximum may not occur at the end of the excavation.

Figure 20 shows the movement (exaggerated 20x) of the soil outside the excavation. Basically, the movement results from rebound due to the unloading. In the exaggerated view, it looks like the rebound is significant, but in actual fact it is relatively small. The maximum at the excavation base is only about 0.1 m (100 mm).

Figure 20. Surface movements.

The rebound along the excavation base is, of course, not evident at the actual site, since the excavators keep removing materials to the design elevation.
Of more significance and interest is the rebound of the ground surface just outside of the wall. Usually, a major concern is the settlement that often occurs behind the retaining wall. The analysis results seem to suggest that it is not an issue. At a first glance, it would seem that the numerical model has not provided the correct response. Upon further reflection, however, it is reasonable that the soil will rebound when it is unloaded. Why does the modeling not match the observed field behavior?

One aspect of shoring wall construction that the modeling does not capture is the loss of ground behind the wall. This can be particularly problematic in a pile-lagging system, where portions of the excavation face are exposed for a period of time before the lagging is installed. Furthermore, there may be some settlement before the lagging picks up the load; that is, slack in the system.

In the case of a carefully constructed diaphragm wall where there is likely little or no loss of ground behind the wall, there may indeed be a slight amount of rebound outside of the wall, but in the field it may be too small to be noticeable.

The apparent uplift is not evident in the numerical results if the excavation base upward pressure relief is ignored in the analysis. Ignoring the up base uplift, however, results in a deflection profile that does not match the measured deflection, as discussed below. To match the measured deflection, it is necessary to consider the base rebound, indicating it should not be ignored.

As a very broad principle in this industry, more expensive shoring systems like diaphragm walls are used in cases where settlement outside the wall is a major concern. Less expensive systems like piles with lagging are used when settlement is less of a concern. The point is that the potential for settlement is related to the shoring system behavior and the installation procedures. The modeling, unfortunately, cannot capture this aspect of the shoring behavior.

From a modeling perspective, the results should not be dismissed because of the small rebound behind the wall. The results related to aspects like lateral deflections and structural stresses are useful in the shoring design.

**Summary and Conclusions**
The two most important aspects of an analysis like this are:

- The numerical modeling techniques and procedures, and
- The soil properties.

The actual wall deflection will be directly related to the soil properties specified. In this case, it was possible to obtain good agreement between the measured and computed wall deflection because the actual deflections were known ahead of time. The material properties could be adjusted until a good agreement was obtained. In actual project work, this is, of course, not the case. In the end, the computed predictions are only as good as the certainty with which the soil properties can be defined, and the analysis results must always be interpreted in this context.
The value in modeling a case like this is not so much in predicting the exact magnitude of the wall deflections, as it is in getting the correct shape and form of the deflection. Being able to predict the correct shape and form of the wall deflection infers a good understanding of the wall behavior and the key issues in the wall performance.

This analysis demonstrates that SIGMA/W has all the features and capabilities necessary to simulate the staged construction of a tie-back retaining wall with pre-stressed anchors.

**References**