Introduction
This example simulates a series of triaxial tests, which can be used to verify that the Mohr-Coulomb and Tresca constitutive models are functioning properly. The simulations include:

1. Unconfined, undrained sample using Tresca;
2. Drained strain-controlled tests;
3. Load-unload-reload tests;
4. Extension tests; and,
5. Undrained strain-controlled tests.

The verification includes comparisons with hand-calculated values and discussions relative to the Mohr-Coulomb theoretical framework.

Numerical Simulation
The problem configuration for the example is shown in Figure 1. The simulated shearing phases are preceded by the simulation of the consolidation phase of a triaxial test (Figure 2). Consolidation is isotropic with the confining pressure equal to 100 kPa, except in analysis A in which a confinement pressure is not applied. The consolidation stage is set as the “Parent”; that is, the initial condition for the subsequent simulations involving shearing.
Figure 1. Triaxial test configuration for establishing initial stress state.

Analyses

- **2D Axisymmetric Geometry**
  - A - Unconfined undrained on Tresca
  - B - Confined drained [100 kPa]
    - a) compression \( \Phi' = 1e-05, c = 50 \)
    - b) compression \( \Phi' = 30, c = 0 \)
    - c) compression \( \Phi' = 30, c = 20 \)
    - d) load-unload-reload \( \Phi' = 30, c = 0 \)
    - e) extension \( \Phi' = 30, c = 0 \)
  - C - Confined undrained [100 kPa]
    - a) compression on Tresca \( Su = 50 \)
    - b) compression \( \Phi' = 30, c = 0 \) [2]

Figure 2. Analysis Tree for the Project.

The shearing phase of the analysis is simulated as a strain-rate controlled test. The definition of the strain-rate involves defining the number of ‘time’ steps and the displacement that occurs over each step. Although the ‘time’ steps are being defined, it is more appropriate to think of the time steps as load steps. Absolute time has no meaning in the context of these analyses. The number of load steps defined in the shear stage simulations is 10 or 50, and the y-displacement (i.e. the boundary condition) at the top of the specimen is defined as a function with a final, total displacement of -0.015 m, where the negative sign indicates downward displacement.
Symmetry is assumed about the vertical and horizontal centre-lines; consequently, only \( \frac{1}{4} \) of the specimen is simulated. The dimensions of the simulation portion of the specimen are 0.025 m by 0.05 m, which is half of the width and height of a conventionally sized triaxial specimen. A total vertical \( y \)-displacement of 0.015 m produces an axial strain of 0.3 (30%).

The GeoStudio file should be consulted to explore the material definition. The first three analyses under ‘B – Confined Drained’ consider compression tests for materials with differing values of effective cohesion and effective friction angle. Analysis B-d) (load-unload-reload) examines the behavior of a frictional soil under loading, unloading and then reloading. This loading path is simulated with a displacement function (Figure 3). The negative sign means the top of the sample is pushed downwards. SIGMA/W is an incremental formulation and, therefore, the applied displacement for each load step is the difference between the function value at the current and previous load steps; that is, it is the incremental change along the function. This means that where the function changes direction, the sign of the applied displacement changes and, consequently, the sample is unloaded.

Analysis B-e) considers an extension test, where the \( y \)-displacement function is in the positive direction. This indicates vertical displacement is upward. Lastly, the Analysis C branch considers a confined undrained scenario using the Tresca and Mohr-Coulomb material models. For the Mohr-Coulomb model, the Response Type is set to Undrained. The Tresca model is automatically set to Undrained.

**Results and Discussion**

**Analysis A – Unconfined, undrained on Tresca**

Figure 4 shows the SIGMA/W simulated stress-strain curve for the unconfined, undrained test using the Tresca material model. The failure criterion for a conventional triaxial test can be written as:
\[
\sigma_1 - \sigma_3 = 2S_u
\]  
\text{Equation 1}

The soil is linear elastic until the stress path reaches the failure line. Afterwards, the soil behavior becomes perfectly plastic. The (triaxial) deviatoric stress \((\sigma_1 - \sigma_3)\) at failure is equal to the maximum vertical stress \(\sigma_1\) (Figure 5). The vertical stress \(\sigma_1\) reaches 100 kPa and the horizontal stress \(\sigma_3\) is zero because this is an unconfined test. Consequently, \(\tau_{\text{max}}\) is equal to the undrained strength (50 kPa).

![Figure 5. Deviatoric stress for unconfined compression test.](image)

**Analysis B-a) – Confined, drained \([\Phi' = 1e-05, c=50]\)**

A confined drained triaxial compression test is simulated on a specimen with a cohesion of 50 kPa. Figure 6 and Figure 7 present the y-effective stress and deviator stress versus axial strain, respectively. The maximum vertical stress increases to 200 kPa because the initial confining stress was 100 kPa. The deviatoric stress \((\sigma_1 - \sigma_3)\) remains the same as that simulated by unconfined undrained test at 100 kPa. The maximum shear stress \(\tau\) is therefore 50 kPa.
Analysis B-b) – Confined, drained [\(\Phi' = 30, c=0\)]

A confined drained triaxial compression test is simulated on a specimen with \(\Phi = 30\). Figure 8 presents the y-effective stress versus axial strain. For a purely frictional soil, the principal stress ratio at the point where the stress level reaches the soil strength is given by:

\[
\frac{\sigma'_1}{\sigma'_3} = \frac{1 + \sin \phi}{1 - \sin \phi}
\]

Equation 2

which is equal to 3 if \(\phi\) is 30 degrees.

The confining stress \(\sigma_3\) is 100 kPa; therefore, \(\sigma'_1\) at failure should be 300 kPa. Note, the starting stress is 100 kPa and the failure stress is 300 kPa. The maximum deviatoric stress is 200 kPa \((300 - 100 = 200\text{ kPa})\), as shown in Figure 9.
Analysis B-c) – Confined, drained [\(\phi' = 30, C = 20\)]

Analysis B-c) is a repeat of the previous case, but now the soil has some cohesive strength (\(c = 20\) kPa). Figure 10 presents the y-effective stress versus axial strain. For a Mohr-Coulomb strength envelope in a triaxial test the vertical stress is given by:

\[
\sigma_1' = \sigma_3' \frac{1 + \sin \phi}{1 - \sin \phi} + \frac{2ccos \phi}{1 - \sin \phi} \quad \text{Equation 3}
\]

The confining stress (\(\sigma_3'\)) is 100 kPa. The maximum \(\sigma_1'\) then will be 369 kPa. The deviatoric stress is 269 kPa (Figure 11).
Analysis B-d) – Load-unload-reload

With the Mohr-Coulomb constitutive soil model, the soil returns to being perfectly elastic when it is unloaded after having gone plastic. This response is illustrated in Figure 11.
Figure 11. Stress-strain during unloading and reloading.

**Analysis B-e) – Extension test**

For a purely frictional soil, the principal stress ratio at the point where the stress level reaches the soil strength is given by:

\[
\frac{\sigma'_1}{\sigma'_3} = \frac{1 + \sin \phi}{1 - \sin \phi}
\]

**Equation 4**

which is equal to 3 if \( \phi \) is 30 degrees. The extension test is simulated by pulling upward on the specimen (refer to the associated boundary condition). In this case, the confining stress is 100 kPa and is the major principal stress \( \sigma'_1 \). The vertical stress is the minor principal stress \( \sigma'_3 \). Therefore, \( \sigma'_3 \) at failure should approach 33.3 kPa (Figure 13). Note, the starting stress is 100 kPa and then drops to 33.3 kPa. The deviatoric stress at the end should be 66.7 kPa, as is shown in Figure 14.

Figure 13. Stress-strain curve for extension test.
Figure 14. Deviatoric stress for extension test.

**Analysis C-a) – Confined, undrained on Tresca**

In conventional undrained triaxial compression test, the total stress path tracks at 3:1 in $q - p'$ space while the mean effective stress path ($q - p'$) is vertical (Figure 15). Stated another way, the mean effective stress remains constant under triaxial loading conditions, which ensures that the elastic volumetric strain is zero. In keeping with Analysis B a), the deviatoric stress at failure is 100 kPa.

Figure 15. Effective $q - p'$ stress path under undrained conditions.

**Analysis C-b) – Confined, undrained [Phi'=30, c=0]**

The mean effective stress path is again vertical in $q - p'$ stress space. Failure is reached at a deviatoric stress of 120 kPa. The y-effective and x-effective stress at failure are 180 kPa and 60 kPa, respectively, making the $\sigma_1'/\sigma_3'$ exactly 3.0 as observed in Case B b).
Summary and Conclusions
This example verifies the response of the Mohr-Coulomb constitutive model in SIGMA/W. Several triaxial tests were simulated and discussed.